Efficient Metafilm/Metasurface Characterization for Obliquely Incident TE Waves via Surface Susceptibility Models

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Abstract — The consistent and precise characterization of metafilms/metasurfaces via a set of electric and magnetic surface susceptibilities is presented in this paper by means of a systematic technique. The new method, which can be regarded as an enhanced generalization of existing approaches, offers extra degrees of freedom and leads to very satisfactory solutions, even for hard-to-model cases. Numerical simulations of various metamaterial unit cells are conducted in order to substantiate the accuracy of the proposed algorithm and confirm the benefits of our formulation.

I. INTRODUCTION

Metafilms or metasurfaces, engineered by arranging electrically small scatterers at a surface, are typically considered as the 2-D equivalent of metamaterials [1], [2]. In fact, they are easier to fabricate compared to bulk metamaterials, as they are smaller in physical size, resulting in lower cost and less lossy structures [3]-[8]. Their merits over the latter inspired applications, like controllable surfaces, miniaturized cavity resonators, waveguides or shielding materials. Lately, it has been proven that homogenization techniques, developed for bulk metamaterials, can lead to the determination of non-unique effective constitutive parameters, when applied to a metafilm which does not have the well-defined thickness of a bulk medium [1]. A metafilm is more appropriately characterized by generalized sheet transition conditions (GSTCs) that relate electromagnetic fields at its interface. Therefore, their accurate and unambiguous description may be only achieved through the calculation of surface susceptibility models.

In this paper, enhanced surface susceptibility relations of metafilms for obliquely incident transverse electric (TE) waves are introduced, thus improving the performance of conventional approaches. The novel methodology extracts the unknown susceptibilities from the *S*-parameters of normally incident TE-plane waves, assuming only the electrically small size of its consisting particles, in order for the point-dipole approximation to acceptably apply. In this manner, a set of robust closed-form expressions of the dynamic interaction constants and the average fields on the interfaces of the metafilm are derived through the appropriate mathematical manipulations. Hence, the reflection and transmission coefficients for an arbitrary obliquely incident TE plane wave can be very accurately predicted from the surface susceptibility model, even for large angles of incidence. Numerical verification involves the analysis of different electric resonators and comparisons with the outcomes of existing algorithms.

II. FORMULATION OF THE PROPOSED METHODOLOGY

Let us assume a metasurface of discrete particles with lattice periods a and b along the x and y direction, respectively, illuminated by a TE-polarized (electric field intensity along x-axis) plane wave propagating on the y-z plane at an angle θ with respect to the z-axis. Supposing that the dimensions of lattice periods are small enough compared to the wavelength, such a structure can be efficiently modeled by replacing its particles with three electric and three magnetic dipoles located at their centers and directed towards the coordinate axes. For the particular problem of TE-polarized waves and the metamaterial cells, studied herein, only x-directed electric as well as y- and z-directed magnetic dipoles are considered. Initially, particle polarizabilities are derived from the *S*-parameters of a normally incident plane wave in terms of [3]. Then, local field expressions for an arbitrary scatterer with dipole moments p_x, m_y, and m_z can be written as

$$E_{x,\mathrm{loc}} = E_0 + \frac{1}{(ab)^{3/2}\varepsilon_0}\left(C^{xx}p_x + \frac{D^{xy}}{c}m_y + \frac{D^{xz}}{c}m_z\right), \quad (1)$$

$$H_{y,\mathrm{loc}} = \frac{E_0}{\eta_0}\cos\theta + \frac{1}{(ab)^{3/2}}\left(cD^{yx}p_x + C^{yy}m_y + C^{yz}m_z\right), \quad (2)$$

$$H_{z,\mathrm{loc}} = -\frac{E_0}{\eta_0}\sin\theta + \frac{1}{(ab)^{3/2}}\left(cD^{zx}p_x + C^{zy}m_y + C^{zz}m_z\right), \quad (3)$$

where c is the speed of light, and C^{ij}, D^{ij}, for $i, j = (x, y, z)$, are the dimensionless dynamic interaction coefficients that specify the contribution of the entire array to the local field of a specific scatterer. As a result of reciprocity ($C^{ij} = C^{ji}$ and $D^{ij} = D^{ji}$) and the symmetries of the problem ($C^{yz} = C^{zy} = 0$ and $D^{xy} = D^{yx} = 0$), the number of unknown coefficients is reduced to four. Due to the process of [4] and after the pertinent algebra, a set of consistent relations for the calculation of these constants is obtained. Recalling that $p_x = \varepsilon_0 a_{Exx} E_{x,\mathrm{loc}}$, $m_y = a_{Myy} H_{y,\mathrm{loc}}$, and $m_z = a_{Mzz} H_{z,\mathrm{loc}}$, the system of equations (1)-(3) is solved and dipole moments are efficiently related to electric a_{Exx} and magnetic a_{Myy}, a_{Mzz} particle polarizabilities.

The summation of the dipole moments, so evaluated, for all the particles comprising the metafilm, results in the average polarization densities that may be associated to surface susceptibilities χ_{ES}, χ_{MS} and the average fields as

$$\bar{P}_{sx} = \frac{p_x}{ab}e^{-jk_0\sin\theta y} = \varepsilon_0\chi_{ES}^{xx}\left(E_x^{inc} + \bar{E}_x^{\mathrm{scat}}\right), \quad (4)$$

$$\bar{M}_{sy} = \frac{m_y}{ab}e^{-jk_0\sin\theta y} = -\chi_{MS}^{yy}\left(H_y^{inc} + \bar{H}_y^{\mathrm{scat}}\right), \quad (5)$$

$$\bar{M}_{sz} = \frac{m_z}{ab}e^{-jk_0\sin\theta y} = -\chi_{MS}^{zz}\left(H_z^{inc} + \bar{H}_z^{\mathrm{scat}}\right), \quad (6)$$

with the bar denoting the average value of the specific quantity and k_0 the magnitude of the wavevector. To compute, the desired surface susceptibilities in (4)-(6), the average scattered fields at the $z = 0^-$ and $z = 0^+$ interface must be evaluated. By substituting the average polarization densities with equivalent

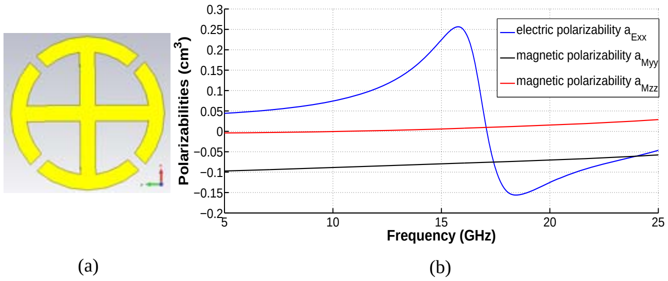

Fig. 1. (a) Geometry of the OE6 ELC resonator and (b) polarizabilities of the resonator (dimensions: unit cell size a = 6.75 mm, outer radius r = 2.5 mm, metal width w = 0.5 mm and gap width g = 0.3 mm.

electric and magnetic surface currents and via the necessary electromagnetic field calculations, these fields are given by

$$\bar{E}_x^{\text{scat}} = -\frac{jk_0}{2\cos\theta\varepsilon_0}\bar{P}_{sx} + \frac{j\omega\mu_0\tan\theta}{2}\bar{M}_{sz}, \tag{7}$$

$$\bar{H}_y^{\text{scat}} = -\frac{jk_0\cos\theta}{2}\bar{M}_{sy}, \tag{8}$$

$$\bar{H}_z^{\text{scat}} = \frac{j\omega\tan\theta}{2}\bar{P}_{sx} - \frac{jk_0\sin\theta\tan\theta}{2}\bar{M}_{sz} \tag{9}$$

Plugging (7)-(9) and the dipole moments calculated from (1)-(3) into (4)-(6) and solving for the surface susceptibilities, the desired description of the metasurface is successfully accomplished. Finally, the transmission and reflection coefficients for an arbitrary obliquely incident wave may be easily estimated from the corresponding expressions provided in [1].

III. NUMERICAL RESULTS – CONCLUSIONS

The validity of the new method is explored in comparison with other well-established algorithms. To this end, the OE6 electric-LC resonator of Fig. 1a is selected and simulated by means of the finite integration technique (FIT) [6]. The electric and magnetic polarizabilities, shown in Fig. 1b, reveal a strong resonance at 17.1 GHz. This resonance is of electric nature, as the name of the structure suggests, owing to its symmetrical shape, which yields an even number of loops.

Next, a 60^{o} incident TE-polarized plane wave is considered in an effort to compare the performance of our algorithm with the simulation results and those obtained through existing schemes. It has to be emphasized that for the technique in [1], the S-parameters from both a normal and an obliquely incident wave (herein an incidence angle of θ = 45^{o} is assumed) are required for the evaluation of surface susceptibilities. As a matter of fact, this direct dependence of the latter on the incidence angle constitutes the major drawback of the particular approach, since the results for any oblique incidence vary with the initial selection of θ. Consequently, there is no optimal choice of θ for the accurate determination of the S-parameters in the case of an arbitrary incidence angle. On the other hand, the method in [2] implements a sparse approximation formula to relate surface to particle polarizabilities and is proven adequate only for small angles of incidence.

Proceeding to the comparison of the reflection, illustrated in Fig. 2, and the transmission coefficient, presented in Fig. 3, one can promptly detect the excellent agreement of our methodology with the simulation results, for a wide frequency range up to 25 GHz. A downshift in the resonance frequency appears for the method presented in [1], while the algorithm in [2] exhibits an evident discrepancy from the simulation results. It is interesting to mention that the proposed technique is proven more accurate in the S-parameters prediction of a 45^{o} incident wave (not shown here), despite the fact that in this case the angle of incidence coincides with the choice of θ for the approach in [1], as previously discussed.

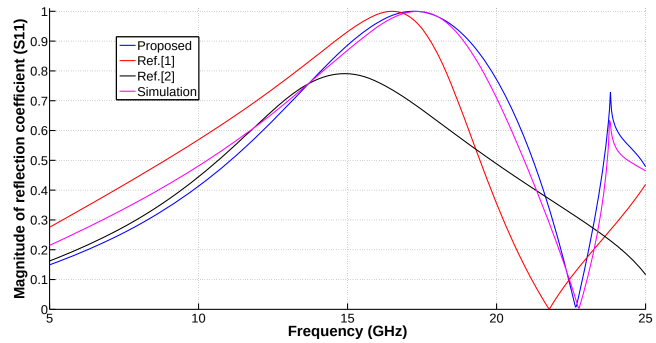

Fig. 2. Comparison between the simulation method and various techniques for the reflection coefficient of a 60^{o} incident plane wave.

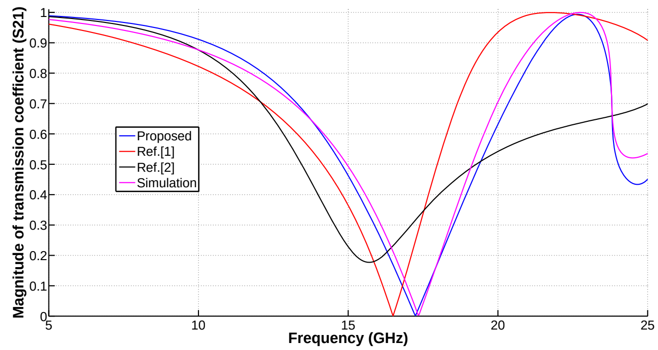

Fig. 3. Comparison between the simulation method and various techniques for the transmission coefficient of a 60^{o} incident plane wave.

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